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Minimization of thermal expansion of symmetric, balanced, angle ply laminates by optimization of fiber path configurations

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ABSTRACT

Optimal fiber path configurations that minimize the sum of the coefficients of thermal expansion (CTE) values along the principal material directions for a class of laminates are presented. Previous studies suggest that balanced, symmetric, angle ply laminates exhibit negative CTE values along the principal directions. Using the sum of the CTE values along the principal material directions as an effective measure of the coefficient of thermal expansion (CTE_{eff}), we have shown and provided a proof that the smallest value of CTE_{eff} is rendered by straight fiber path configurations. The laminates considered are sufficiently thin so as to neglect the thermal stresses induced through the thickness of the laminate. It is found that the minimal CTE_{eff} values occur for $[+45/-45]_{ns}$ lay-ups. This result is supported by numerical studies that consider curvilinear fiber paths. The possibility of obtaining zero CTE values along both principal material directions and the conditions that render this situation are also examined.

1. Introduction

Failure by thermal fatigue can be mitigated by minimizing the coefficient of thermal expansion (CTE) of composite laminates α_X^L and α_Y^L along the principal material directions. Early work on characterization of CTE values in fiber reinforced composites was due to Craft and Christensen [1], Marom and Weinberg [2], Ishikawa and Chou [3], Bowles and Tompkins [4], Sleight [5], Lommens et al. [6], and references therein. More recent studies have focused on laminates with straight fiber configurations [7–10]. Amongst these, those which are balanced, symmetric, angle ply ($[+\theta/-\theta]_{ns}$) lay-ups have been found to exhibit anomalous mechanical response. Analysis of these laminates have shown the existence of negative CTE (α_X^L, α_Y^L) for certain range of ply orientations [7,9]. Zhu and Sun [10], showed that the ratio of shear modulus G_{12} to the Young's modulus E_1 is an important parameter that determines the sign and magnitude of the CTE in the composite laminate. However, negative CTE values along both principal material directions (x, y) of the composite laminate were not obtained simultaneously for any ply angle θ. In this study, we relax the requirement of fibers having straight configurations and seek the optimum fiber path that yields the least value of CTE_{eff}, maintaining the assumption of a balanced, symmetric, angle ply laminate. In the analysis to follow, the possibility of obtaining a zero value for CTE_{eff} is investigated and conditions for obtaining such a CTE_{eff} are derived.

2. Model description

Consider curvilinear fiber configurations in the x–y plane which are symmetric about the z-axis (Fig. 1) in the Representative Unit Cell (RUC) of in-plane dimensions $A \times B$. The fibers are stacked parallel to the y-axis. Obliquely stacked configurations of the fibers are not considered since it reduces to the case under consideration as can be seen from Fig. 2. This would imply that for any infinitesimally small portion of the fiber curve with an orientation θ, there exists an infinitesimally small complementary fiber element with orientation $-\theta$ (Fig. 3). For every fiber at angle $+\theta$ at $z = +z^*$, there is another fiber of same orientation at $z = -z^*$. Also, for every fiber at an angle $-\theta$ at $z = +z^{**}$, there is another fiber at $z = -z^{**}$ with the same orientation. Hence, this configuration acts as a balanced symmetric laminate for which the moment resultants due to thermal stresses cancel out (see [11]), i.e. $M_x^* = 0, M_y^* = 0$ and $M_{xy}^* = 0$. Here, we use standard composite laminate nomenclature as given in [11]. Similarly, the effective shear force resultants due to thermal expansion also cancel out, i.e. $N_{xy}^* = 0$. Therefore, the only non-zero stress resultants present are normal stresses along the principal directions $\left(N_x^* \text{ and } N_y^*\right)$ in the plane of the laminate. The

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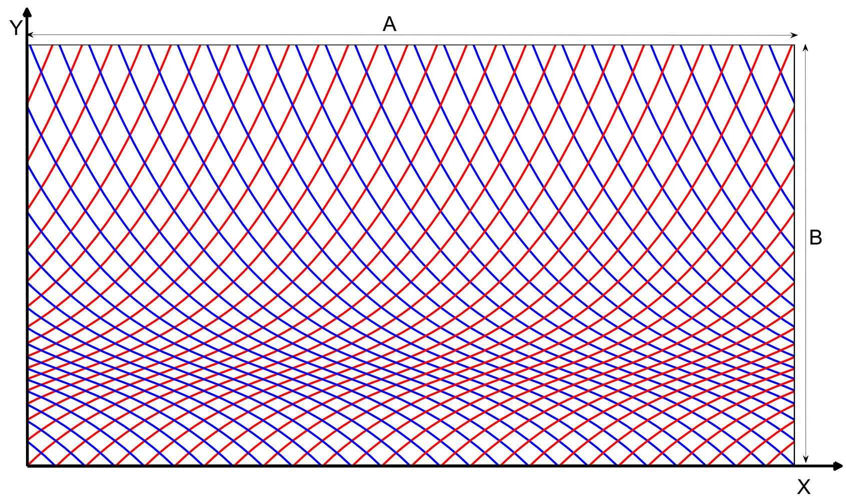

Fig. 1. Profile of fibers in the Representative Unit Cell (RUC) of dimensions $A \times B$. The RUC has many overlaid symmetric fibers which renders the RUC to have a structure similar to that of a balanced, symmetric, angle ply laminate.

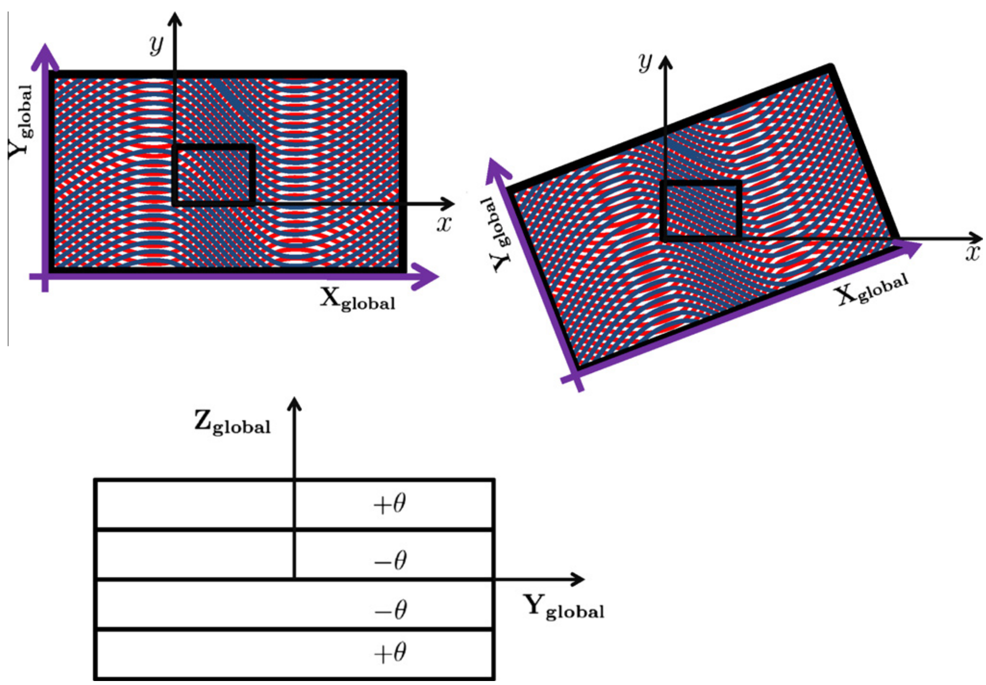

Fig. 2. Schematic showing the equivalence of the fiber stacking along the horizontal and oblique directions.

curvilinear fiber format in the RUC is invariant along the *y*-direction. Hence, the compliance matrix of an infinitesimal strip of width *dx* is a function of *x* alone. The continuity of fiber slopes across adjacent RUCs is ensured by the equality of slope at RUC boundaries.

3. Mathematical formulation

3.1. Straight Fibers

For a straight fiber, balanced, angle ply laminate, if the fiber is oriented at an angle θ with respect to *x*-direction [7,10], we have

$$\alpha_X^L = \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta + \frac{\bar{S}_{16}}{\bar{S}_{66}}(\alpha_2 - \alpha_1)\sin 2\theta \tag{1}$$

$$\alpha_Y^L = \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta + \frac{\bar{S}_{26}}{\bar{S}_{66}}(\alpha_2 - \alpha_1)\sin 2\theta \tag{2}$$

where

$$\begin{aligned} \bar{S}_{16} &= \{2(S_{11} - S_{12}) - S_{66}\}\cos^3\theta\sin\theta \\ &\quad + \{2(S_{12} - S_{22}) + S_{66}\}\cos\theta\sin^3\theta \\ \bar{S}_{26} &= \{2(S_{11} - S_{12}) - S_{66}\}\cos\theta\sin^3\theta \\ &\quad + \{2(S_{12} - S_{22}) + S_{66}\}\cos^3\theta\sin\theta \\ \bar{S}_{66} &= 2\{2(S_{11} + S_{22} - 2S_{12}) - S_{66}\}\cos^2\theta\sin^2\theta \\ &\quad + S_{66}\{\cos^4\theta + \sin^4\theta\} \end{aligned} \tag{3}$$

For no thermal expansion, we should have $\alpha_X^L = 0$ and $\alpha_Y^L = 0$ simultaneously. This leads to $(\alpha_X^L - \alpha_Y^L) = 0$.

$$(\alpha_1 - \alpha_2)\left\{\cos 2\theta - \left(\frac{\bar{S}_{16} - \bar{S}_{26}}{\bar{S}_{66}}\right)\sin 2\theta\right\} = 0 \tag{4}$$

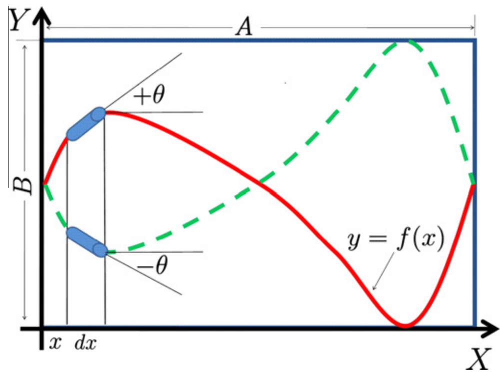

Fig. 3. Profile of a pair of complementary fibers in the Representative Unit Cell (RUC) of dimensions $A \times B$ with fiber curve (red) modeled as a function $y = f(x), x \in [0, A]$. The complementary fiber curve (green) is also shown. The figure shows the definition of the angle θ for an infinitesimal element of the fiber. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In the above expression, $\alpha_1 \neq \alpha_2$. We inspect the term in the parenthesis which is expanded as

$$\cos 2\theta \left\{ 1 - \frac{\sin^2\theta (S_{11} + S_{22} - 2S_{12} - S_{66})}{(S_{11} + S_{22} - 2S_{12})\sin^2 2\theta + S_{66}\cos^2 2\theta} \right\} = 0 \quad (5)$$

By inspection, $\theta = \pi/4$ is a solution to Eq. (5). Adding Eqs. (1) and (2) we get

$$\alpha_X^L + \alpha_Y^L = (\alpha_1 + \alpha_2) + (\alpha_2 - \alpha_1)\left(\frac{\bar{S}_{16} + \bar{S}_{26}}{\bar{S}_{66}}\right)\sin 2\theta = 0 \quad (6)$$

Setting $\theta = \pi/4$, the above expression is written as

$$\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} = \left\{ \frac{S_{11} - S_{22}}{S_{11} + S_{22} - 2S_{12}} \right\} \quad (7)$$

Writing the compliance quantities in terms of elastic constants, we have $S_{11} = 1/E_1$, $S_{22} = 1/E_2$, and $S_{12} = -\nu_{12}/E_1$. Therefore,

$$\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} = \left\{ \frac{\frac{1}{E_1} - \frac{1}{E_2}}{\frac{1}{E_1} + \frac{1}{E_2} + 2\frac{\nu_{12}}{E_1}} \right\} \quad (8)$$

Since $E_1 > E_2$, right hand side (RHS) $< 0 \Rightarrow (\alpha_1 - \alpha_2) < 0 \Rightarrow \alpha_1 < \alpha_2$. Next, assuming $(\alpha_1 + \alpha_2) > 0$

$$\frac{\alpha_1}{\alpha_2} = -\left\{ \frac{\frac{1}{E_1} + \frac{\nu_{12}}{E_1}}{\left(\frac{1}{E_2} + \frac{\nu_{12}}{E_1}\right)} \right\} < 0 \quad (9)$$

A similar result has also been presented in Zhu and Sun [10]. Using Eqs. (8) and (9),

$$\alpha_1 < 0 < \alpha_2 \quad (10)$$

3.2. Curved fibers

Now, consider a single curvilinear fiber path in the x-y plane and represent it by a function $y = f(x)$. If θ is the angle made by any infinitesimally small segment of the curve with the x-axis, the sine and cosine terms for this segment can be written in terms of the slope of the curve $y' = \frac{df(x)}{dx} = \tan\theta$.

$$\cos\theta = \frac{1}{\sqrt{1 + (y')^2}} = c, \quad \sin\theta = \frac{y'}{\sqrt{1 + (y')^2}} = s \quad (11)$$

The compliance matrix $[\hat{S}]$ terms are obtained from

$$\hat{S}_{16} = \frac{\int_0^B \int_0^A \bar{S}_{16}dxdy}{AB}, \quad \hat{S}_{26} = \frac{\int_0^B \int_0^A \bar{S}_{26}dxdy}{AB}, \quad \hat{S}_{66} = \frac{\int_0^B \int_0^A \bar{S}_{66}dxdy}{AB} \quad (12)$$

Since the fibers are stacked along the y-direction in the RUC, there is no variation of $\hat{S}_{16}, \hat{S}_{26}$ and $\hat{S}_{66}$ along the y-direction. Thus, we make the assumption of constant strain along the x-axis of the RUC.

Hence, the compliance matrix $[\hat{S}]$ terms reduce to

$$\hat{S}_{16} = \frac{\int_0^A \bar{S}_{16}dx}{A}, \quad \hat{S}_{26} = \frac{\int_0^A \bar{S}_{26}dx}{A}, \quad \hat{S}_{66} = \frac{\int_0^A \bar{S}_{66}dx}{A} \quad (13)$$

where the compliance terms of the infinitesimal segment are given by

$$\begin{aligned} \bar{S}_{16} &= \{2(S_{11} - S_{12}) - S_{66}\}c^3s + \{2(S_{12} - S_{22}) + S_{66}\}cs^3 \\ \bar{S}_{26} &= \{2(S_{11} - S_{12}) - S_{66}\}cs^3 + \{2(S_{12} - S_{22}) + S_{66}\}c^3s \\ \bar{S}_{66} &= 2\{2(S_{11} + S_{22} - 2S_{12}) - S_{66}\}c^2s^2 + S_{66}\{c^4 + s^4\} \end{aligned} \quad (14)$$

The CTE in the principal material directions of the laminate are obtained as [21],

$$\begin{aligned} \alpha_X^L &= \frac{1}{A}\int_0^A (\alpha_1 c^2 + \alpha_2 s^2)dx - \frac{\int_0^A \bar{S}_{16}dx}{\int_0^A \bar{S}_{66}dx}\left\{\frac{1}{A}\int_0^A 2cs(\alpha_1 - \alpha_2)dx\right\} \\ \alpha_Y^L &= \frac{1}{A}\int_0^A (\alpha_1 s^2 + \alpha_2 c^2)dx - \frac{\int_0^A \bar{S}_{26}dx}{\int_0^A \bar{S}_{66}dx}\left\{\frac{1}{A}\int_0^A 2cs(\alpha_1 - \alpha_2)dx\right\} \end{aligned} \quad (15)$$

Define a scaled measure of CTE_{eff}, $G = A(\alpha_X^L + \alpha_Y^L)$.

$$G = A(\alpha_1 + \alpha_2) + 4(S_{22} - S_{11})\frac{\left(\int_0^A csdx\right)^2}{\left(\int_0^A \bar{S}_{66}dx\right)}(\alpha_1 - \alpha_2) \quad (16)$$

From Eq. (16),

$$G = A\{(\alpha_1 + \alpha_2) + K_2(\alpha_1 - \alpha_2)\} \quad (17)$$

As $S_{11} = 1/E_1$, $S_{22} = 1/E_2$ and $S_{12} = -\nu_{12}/E_1$, we get

$$\bar{S}_{66} = 2\left[2\left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}\right)\right]c^2s^2 + \frac{1}{G_{12}}(c^2 - s^2)^2 > 0 \quad (18)$$

Also,

$$(S_{22} - S_{11}) = \left(\frac{1}{E_2} - \frac{1}{E_1}\right) > 0. \quad (19)$$

We write G in the manner $G = A(\alpha_1 + \alpha_2) + K_2 A(\alpha_1 - \alpha_2)$, where

$$K_2 = 4\left(\frac{S_{22} - S_{11}}{A}\right)\frac{\left(\int_0^A csdx\right)^2}{\left(\int_0^A \bar{S}_{66}dx\right)} \quad (20)$$

From Eqs. (18) and (19) it follows that

$$K_2 \geqslant 0 \quad (21)$$

3.2.1. Inspecting the behavior of K_2

We inspect if $K_2 < 1$ or $K_2 > 1$. Suppose $K_2 > 1$, then

$$\begin{aligned} &4\left(\frac{S_{22} - S_{11}}{A}\right)\left(\int_0^A csdx\right)^2 - 4\int_0^A (S_{11} + S_{22} - 2S_{12})(cs)^2dx \\ &\quad - \int_0^A S_{66}(c^2 - s^2)^2dx > 0 \end{aligned} \quad (22)$$

Invoke an integral inequality, which is obtained as a special case of the Cauchy–Schwarz inequality,

$$\frac{1}{A}\left(\int_0^A f(x)dx\right)^2 \leqslant \int_0^A (f(x))^2dx, \quad f(x) = cs \quad (23)$$

Hence,

$$4(S_{22}-S_{11})\left(\int_0^A (cs)^2dx\right) - \int_0^A 4(S_{11}+S_{22}-2S_{12})(cs)^2dx - \int_0^A S_{66}(c^2-s^2)^2dx > 0 \quad (24)$$

$$\int_0^A (-8S_{11}+8S_{12})(cs)^2dx - \int_0^A S_{66}(c^2-s^2)^2dx > 0 \quad (25)$$

Substituting the compliance terms with material constants, we get,

$$-8\int_0^A \left(\frac{v_{12}+1}{E_1}\right)(cs)^2dx - \int_0^A \frac{1}{G_{12}}(c^2-s^2)^2dx > 0 \quad (26)$$

Now, assuming $(v_{12} + 1) > 0$, Eq. (26) presents a contradiction as the terms on the left hand side (LHS) are negative. Hence by reductio ad absurdum, $K_2 \leqslant 1$.

From Eq. 17 we can see that G is an increasing function of α_1 and α_2 and $\frac{dG}{d\alpha_1} > \frac{dG}{d\alpha_2}$. Hence, G is minimum when K_2 is maximum and $\alpha_2 > \alpha_1$, or when K_2 is minimum and $\alpha_1 > \alpha_2$.

For zero thermal expansion, $\alpha_X^L = \alpha_Y^L = 0 \Rightarrow G = 0$,

$$(\alpha_1+\alpha_2) + K_2(\alpha_1-\alpha_2) = 0 \quad (27)$$

$$\frac{\alpha_1+\alpha_2}{\alpha_1-\alpha_2} = -K_2 \leqslant 0 \quad (0 \leqslant K_2 \leqslant 1) \quad (28)$$

$$\Rightarrow \alpha_1 < \alpha_2 \quad (29)$$

$$\frac{\alpha_1}{\alpha_2} = \frac{K_2-1}{K_2+1} < 0 \quad (30)$$

$$\Rightarrow \alpha_1 < 0 < \alpha_2 \quad (31)$$

4. Optimal fiber configurations

4.1. Straight fiber

We start with an assumption $\alpha_2 > \alpha_1$. The value of CTE_{eff} is minimum when K_2 is maximum.

$$K_2 = \frac{4(S_{22}-S_{11})c^2s^2}{(4(S_{11}+S_{22}-2S_{12})c^2s^2 + S_{66}(c^2-s^2)^2)} = \frac{S_{22}-S_{11}}{(S_{11}+S_{22}-2S_{12}) + S_{66}\cot^2 2\theta} \quad (32)$$

K_2 is maximum when $\theta = \frac{\pi}{4}$.

$$K_2^{Maximum} = \frac{S_{22}-S_{11}}{(S_{11}+S_{22}-2S_{12})} \quad (33)$$

Now, considering $\alpha_2 > \alpha_1$, the value of CTE_{eff} is minimum when K_2 is minimum. We have already shown the lower bound for K_2 to be 0 (Eq. (21)). It can be seen that when $\theta = 0$, the minimum value of K_2 occurs.

4.2. Curved fiber

We again start with the same assumption as in the previous section , i.e., $\alpha_2 > \alpha_1$. The value of CTE_{eff} is minimum when K_2 is maximum.

$$K_2 = \frac{4(S_{11}-S_{22})\left(\int_0^A cs dx\right)^2}{A\int_0^A \{4(S_{11}+S_{22}-2S_{12})(cs)^2 + S_{66}(c^2-s^2)^2\}dx} \quad (34)$$

Fiber paths that maximize K_2 are sought. As shown in Appendix **A**, among all possible paths, those that satisfy the condition $y' = 1$ are the paths that maximize K_2. This corresponds to straight fiber paths. Furthermore, alternately stacked plies are orthogonal in this configuration.

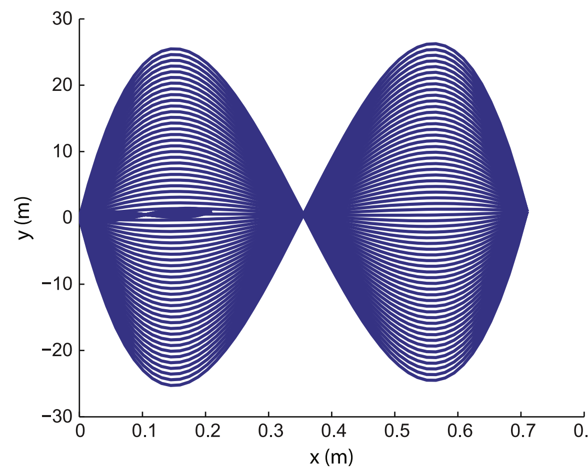

Fig. 4. Schematic illustrating the range of sample candidate cubic polynomials for modeling fiber paths.

Now, considering $\alpha_2 > \alpha_1$, the value of CTE_{eff} is minimum when K_2 is minimum. We have already shown the lower bound for K_2 to be 0 (Eq. (21)). It can be seen that when $y' = 0$, i.e when the fibers in all the plies are in the same orientation, the minimum value of K_2 occurs.

To validate these analytical findings, a numerical study was carried out by modeling the curvilinear fiber configurations using cubic polynomials. The curved fiber paths are selected so as to represent the majority of fiber orientations. The RUC domain under consideration varies from 0.01 × 0.01 sq.m to 1 × 1 sq.m. A sample from the curved fiber paths used for the study is shown in Fig. 4.

The numerical simulations show that the thermal expansivity along each of the principal axis directions is the least when the fiber configuration is approximately linear, i.e. straight fiber configurations. The material used for the numerical study is a glass polypropylene composite having $E_1 = 34.5 \times 10^9$ Pa, $E_2 = 3.1 \times 10^9$ Pa, $v_{12} = 0.25$ $\alpha_1 = 6 \times 10^{-6}$/°C and $\alpha_2 = 100 \times 10^{-6}$/°C. The study shows that the minimum value of α_X^L is obtained for a straight fiber configuration. Similarly, the minimum value of α_Y^L is also obtained for a straight fiber configuration, which is complementary to the fiber configuration at which α_X^L was a minimum. Furthermore, when both α_X^L and α_Y^L are combined such that the area expansion is minimized (in-plane dilatation) to find an optimal path, again an approximate straight fiber path is obtained. These findings compare well with analytical results presented in Ito et al. [7]. Further details of our study is contained in Rangarajan et al. [12].

5. Conclusions

We have shown and provided a proof that amongst all curvilinear fiber configurations, the fiber configuration with straight fiber paths yields the least value of CTE_{eff} for symmetric, balanced, angle ply laminates. This configuration is independent of lamina principal material parameters, E_1, E_2, v_{12}, G_{12}, α_1 and α_2. Additionally, bounds for the values of α_1 and α_2 are presented which can lead to the values of CTE along the principal material directions (α_X^L, α_Y^L) to be zero simultaneously. This implies that there would be no change in the in-plane dimensions (along both principal laminate axes) of the laminates on applying thermal loads, if Eq. (9) is satisfied. The bounds also show that α_1 has to be negative and α_2 has to be positive in order to obtain zero thermal expansion along both the laminate principal axes simultaneously. Amongst the straight fiber path configurations, laminates with $[+45/-45]_{ns}$ lay-up have the least measure of CTE_{eff}. In such cases, the laminate has isotropic values of CTE in the plane of the laminate. The degree to which CTE_{eff} can be minimized is a function of the material parameters mentioned above.

Appendix A

A.1. Finding fiber paths to maximize K_2

$$K_2 = 4\left(\frac{S_{22} - S_{11}}{A}\right)\frac{\left(\int_0^A cs dx\right)^2}{\left(\int_0^A \bar{S}_{66} dx\right)} \quad (35)$$

Replacing c and s in terms of y' from Eq. (11) and using the Cauchy–Schwarz inequality as in Eq. (23), we get

$$K_2 = 4\left(\frac{S_{22} - S_{11}}{A}\right) \times \frac{\left(\int_0^A \frac{y'}{1+(y')^2} dx\right)^2}{\left(\int_0^A \left[\{4(S_{11}+S_{22}-2S_{12})\}\left(\frac{y'}{1+(y')^2}\right)^2 + S_{66}\left\{\left(\frac{1-(y')^2}{1+(y')^2}\right)^2\right\}\right] dx\right)} \quad (36)$$

$$\leqslant 4\left(\frac{S_{22} - S_{11}}{1}\right) \times \frac{\int_0^A \left(\frac{y'}{1+(y')^2}\right)^2 dx}{\left(\int_0^A \left[\{4(S_{11}+S_{22}-2S_{12})\}\left(\frac{y'}{1+(y')^2}\right)^2 + S_{66}\left\{\left(\frac{1-(y')^2}{1+(y')^2}\right)^2\right\}\right] dx\right)} \quad (37)$$

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