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IE 310 Operations Research
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Winter, 2003
Northwestern University

Introduction to Operations Research

QUIZ 1 – SOLUTIONS

Problem 1 (60 percent):

You are in the business of making furniture. Your firm has 3 different products: bookcases, tables and dressers. The revenue that you get from each bookcase is $400, from each table is $750, and from each dresser is $1200. Each item of furniture is composed of some pine and some maple wood. The number of square feet of each type of wood are given below for the three different products.

Item	Pine	Maple
Bookcase	24	30
Table	40	10
Dresser	35	50

You have two sources for each of the two types of wood. The cost per square foot as well as the weekly capacity of each supplier is given below:

Supplier	Pine ($/sq ft)	Maple ($/sq ft)
Nearby	1.35	2.95
Remote	1.75	3.55

Supplier	Pine (Sq ft/wk)	Maple (Sq ft/wk)
Nearby	500	800
Remote	600	1000

Finally, it takes you a certain amount of person-hours to finish each piece. The table below gives these values. Your firm employs enough people to have 2000 person-hours available each week. Each person-hour costs you $8.50.

Item	Person hours required
Bookcase	25
Table	40
Dresser	75

Your objective is to maximize the contribution from producing the various items subject to the relevant constraints.

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Let us define the following indices and variables.

Indices:

b	bookcases
t	tables
d	dressers
n	near
f	far

Variables:

X_b, X_t, X_d	number of bookcases, tables and dressers produced respectively
P_n, P_f	number of square feet of PINE purchased from the near and far suppliers respectively
M_n, M_f	number of square feet of MAPLE purchased from the near and far suppliers respectively
L	number of person hours used each week

a) Write out the objective function in terms of the constants given above and the decision variables we have defined.

Minimize $400X_b + 750X_t + 1200X_d - 1.35P_n - 1.75P_f - 2.95M_n - 3.55M_f - 8.5L$

b) One class of constraints says that the amount of each resource used (pine, maple and labor) must be less than or equal to the amount available. Write out these constraints in terms of the decision variables above. *Note that you do NOT know a priori how much pine or maple you have since the amount that you purchase of each type of wood from each supplier iis a decision variable. You should probably also treat labor the same way (depending on how you wrote the objective function in (a)). In other words, be careful…*

$$24X_b + 40X_t + 35X_d - P_n - P_f \le 0$$
$$30X_b + 10X_t + 50X_d - M_n - M_f \le 0$$
$$25X_b + 40X_t + 75X_d - L \le 0$$

c) A second class of constraints limits the amount of each type of wood you can get from each supplier. Write out these 4 constraints as well as the constraint that limits the amount of labor that you use.

$$\begin{array}{lllllr} P_n & & & & & \le 500 \\ & P_f & & & & \le 600 \\ & & M_n & & & \le 800 \\ & & & M_f & & \le 1000 \\ & & & & L & \le 2000 \end{array}$$

d) In the optimal solution to the problem, we make **no** bookcases, 7.8125 tables and 22.5 dressers each week. We buy 500 square feet of pine from the nearby supplier and 600 square feet of pine from the remote dealer; we also buy 800 square feet of maple from the nearby dealer and 403.125 square feet of maple from the remote dealer. Finally, we use all 2000 person-hours of available production time. The weekly contribution is $7,312 (rounded to the nearest dollar)

Characterize the optimal solution to the problem in terms of a production plan that a manager could use in the event that some of the data were slightly off.

Note that the two binding constraints are the ones on the total amount of pine and the total amount of labor. These constraints boil down to $\begin{array}{l} 40X_t + 35X_d = 1100 \\ 40X_t + 75X_d = 2000 \end{array}$. If these two constraints are solved simultaneously we get the solution given above (7.8125 tables and 22.5 dressers). Thus, the solution is to utilize all the available labor and pine making tables and dressers to satisfy the **pair** of equations above and to buy enough maple to make the indicated number of tables and dressers. No bookcases are to be made.

e) What is the value of the dual variable associated with the constraint on the amount of maple that can be purchased from the remote dealer? Why?

0 since the constraint on the amount of maple purchased from the remote dealer is not binding.

f) If you could buy one more square foot of pine from the more remote supplier, how would the number of dressers produced and the number of tables produced change?

In this case, the solution would be given by solving the following two simultaneous equations: $\begin{array}{l} 40X_t + 35X_d = 1101 \\ 40X_t + 75X_d = 2000 \end{array}$ which gives $X_d = 22.5 - \frac{1}{40}$, $X_t = 7.8125 + \frac{3}{64}$.

g) Would your answer to (f) be different if, instead of buying the additional square foot from the more remote supplier, you could now buy one more square foot from the nearby supplier? Justify your answer briefly. Is there anything about the solution to the problem described in (g) that would be different from the solution to the problem described in (f)?

The solution would not change in terms of the decision variables from that shown in (f), however, the objective function would go up by $0.40 since we are now buying from the closer supplier at a savings of $0.40 per square foot.

Problem 2 (40 percent):

Consider the problem faced by General Motors in helping the railroads to redistribute empty rail cars. In this problem, there are five cities that end up with a surplus of rail cars and three cities that have a net deficit of rail cars each day. The data for the problem, including the distances between each relevant pair of cities, are given in the table below:

	Shortage cities			
Surplus cities	Detroit	Los Angeles	St. Louis	***Net Surplus***
Chicago	238	1746	259	***15***
New York	2456	488	677	***26***
Atlanta	1940	600	468	***25***
Houston	1107	1380	680	***17***
San Diego	1962	116	1556	***45***
Net Shortage	***63***	***27***	***38***	

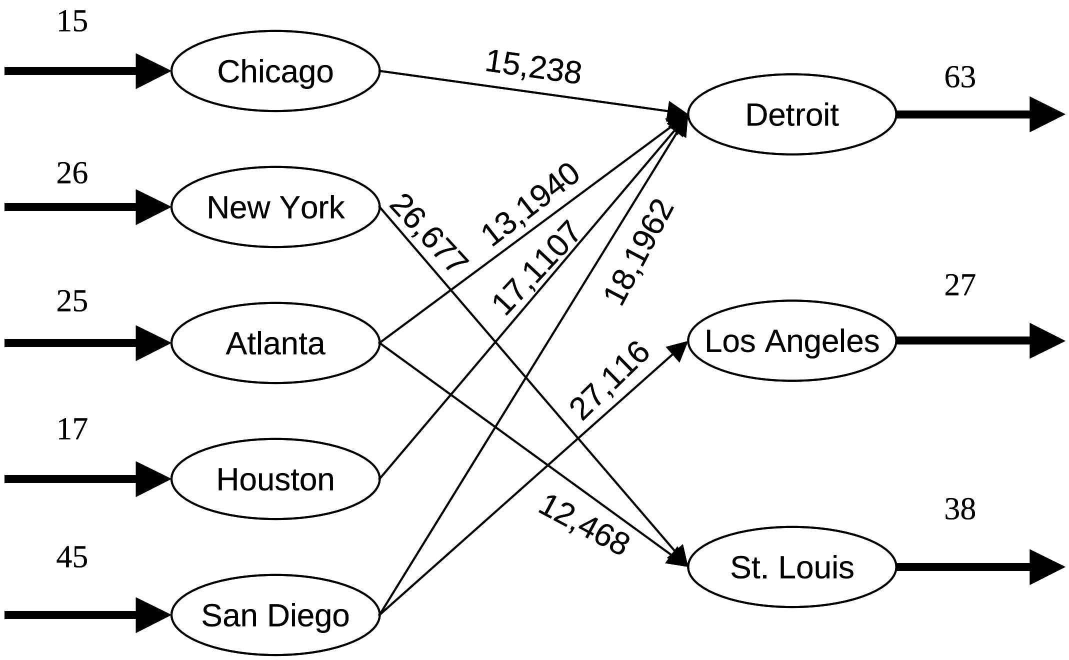
Thus, there are 15 extra rail cars in Chicago each day and the distance between Chicago and Detroit is 238 miles. Similarly, there is a net shortage of 63 rail cars in Detroit each day and the distance between Detroit and San Diego is 1962 miles. Note that the sum of the net surplus is 128 as is the sum of the net shortage.

After minimizing the total distance, we find the following optimal solution:

	Shortage cities					
Surplus cities	Detroit	Los Angeles	St. Louis	Actual flow out		***Reqd Out***
Chicago	15	0	0	15	<=	***15***
New York	0	0	26	26	<=	***26***
Atlanta	13	0	12	25	<=	***25***
Houston	17	0	0	17	<=	***17***
San Diego	18	27	0	45	<=	***45***
Actual flow in	63	27	38			
	>=	>=	>=			
Reqd In	***63***	***27***	***38***			

Total distance **109275**

a) Complete the following diagram showing (i) the allowable flows into each "supply" node, (ii) the required flows out of each "demand" node, (iii) the **actual** flows between each supply and demand node and (iv) the distances between shortage (supply) cities and surplus (demand) cities. Do **<u>not</u>** show flows that are zero. *Note that the values for the flows (15) and distances (238) between Chicago and Detroit are shown in the way they should be displayed.*



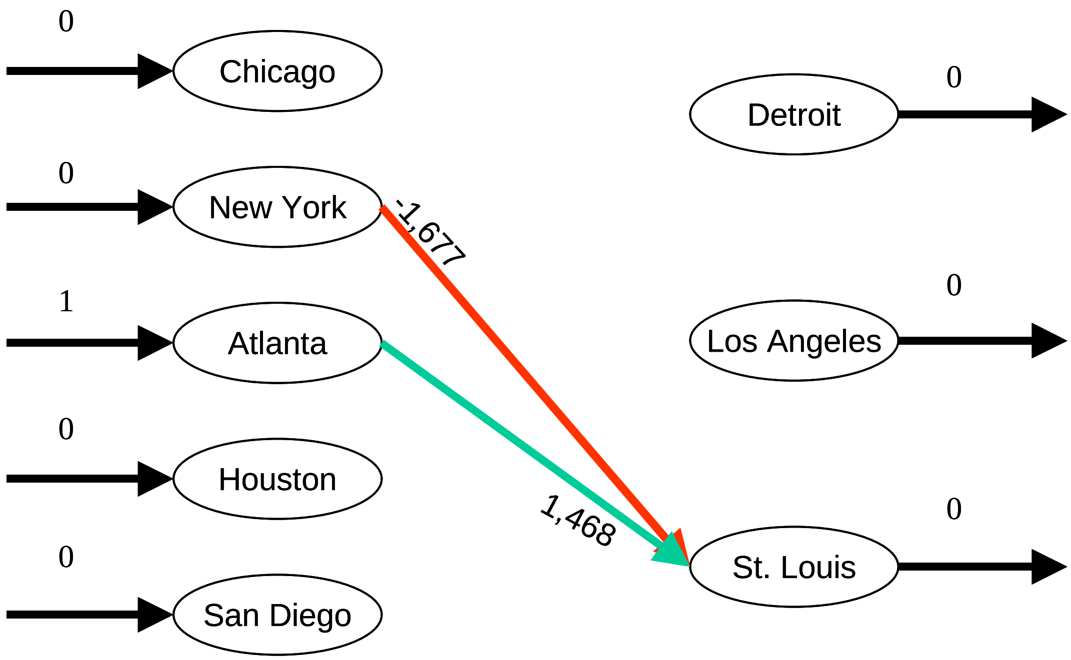
b) The dual variables (shadow prices) for the solution are shown in the table below:

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G27	Chicago	15	-1,911	15	13	0
G28	New York	26	0	26	1E+30	0
G29	Atlanta	25	-209	25	26	0
G30	Houston	17	-1,042	17	13	0
G31	San Diego	45	-187	45	13	0
D32	Detroit	63	2,149	63	0	13
E32	Los Angeles	27	303	27	0	13
F32	St. Louis	38	677	38	0	26

Using the figure from part (a), justify the shadow price of –209 for the supply at Atlanta. In other words, if there were one more rail car available in Atlanta, show how the flows would change and verify that the net change in the objective function is –209. *Note that the total supply will **<u>not</u>** equal the total demand in this part of the question.*

FROM	TO	Change in Flow	Change in cost
Atlanta	St. Louis	+1	468
New York	St. Louis	-1	-677
Net Change			***-209***



Note that the supply at New York does not change, but we use 1 less unit of it

c) Using the figure from part (a), justify the combined shadow prices of –209 for the supply at Atlanta and +303 for the demand in Los Angeles. In other words, show how the flows would change and verify that the combined shadow prices give the net change in the objective function (total rail car miles).

FROM	**TO**	**Change in Flow**	**Change in cost**
San Diego	Los Angeles	+1	116
San Diego	Detroit	-1	-1962
Atlanta	Detroit	+1	1940
Net change			***94=303-209***

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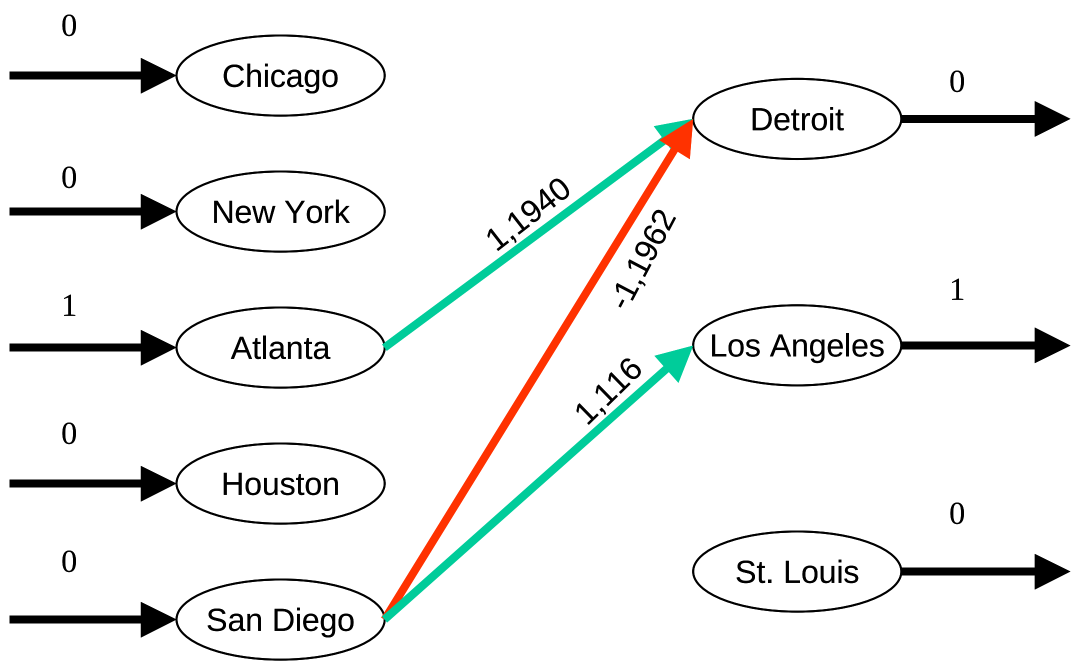

d) Finally, in the table below, we show a **portion** of the reduced cost table for the problem. The reduced cost for the flow from Chicago to Los Angeles is 3354. Show how the flows would change if the solution required one rail car to be moved from Chicago to Los Angeles. Verify that the value of 3354 makes sense.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D27	Chicago Detroit	15	0	238	1,493	1E+30
E27	Chicago Los Angeles	0	3,354	1,746	1E+30	3,354
F27	Chicago St. Louis	0	1,493	259	1E+30	1,493

FROM	TO	Change in Flow	Change in cost
Chcago	Los Angeles	+1	1746
Chicago	Detroit	-1	-238
San Diego	Detroit	+1	1962
San Diego	Los Angeles	-1	-116
Net change			***3354=Reduced cost***

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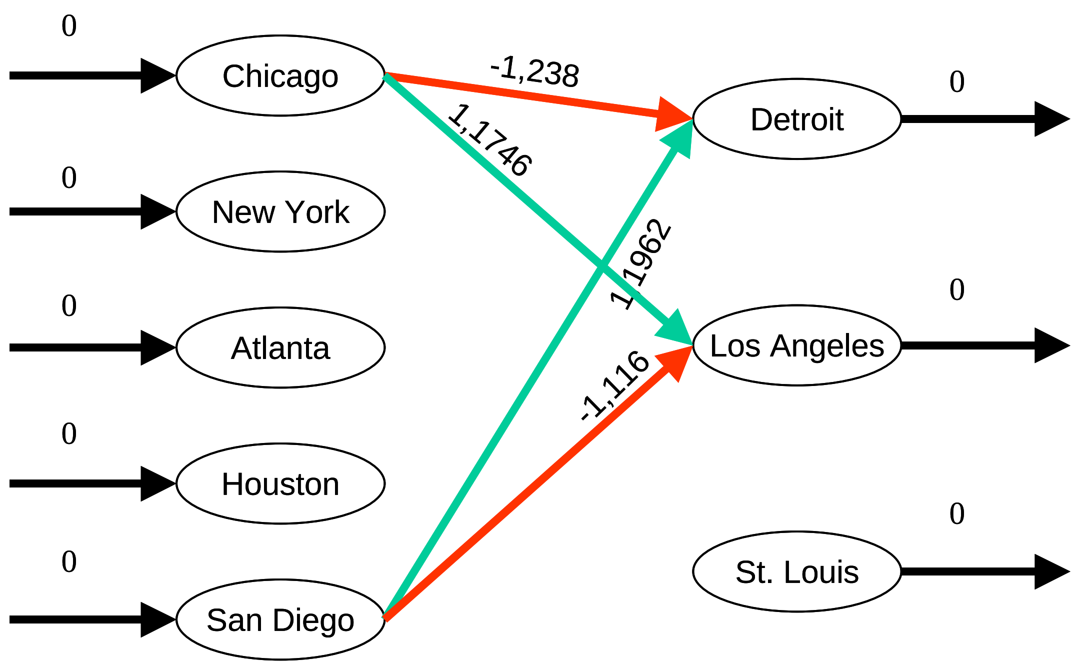

Note: The key in all of (b), (c) and (d) is to look at the flows that you have and to see which ones will change while ensuring that you continue to satisfy the supply and demand constraints.