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Statistics I

Quiz 2

<u>Problem 1:</u> (25 percent)

A sample of 16 faculty members who purchased a new car in the last 12 months was taken to determine the price that the faculty members typically pay for a car. The following information was obtained:

$$\sum_{i=1}^{16} P_i = 339{,}600 \qquad \sum_{i=1}^{16} P_i^2 = 7{,}307{,}083{,}500$$

From this we can compute $\overline{P} = \dfrac{\sum_{i=1}^{16} P_i}{16} = \dfrac{339{,}600}{16} = 21{,}225$ and

$$S^2 = \frac{\sum_{i=1}^{16} P_i^2 - \left(\left(\sum_{i=1}^{16} P_i \right)^2 \Big/ 16 \right)}{15} = \frac{7{,}307{,}083{,}500 - \left(330{,}600^2/16\right)}{15} = 6{,}604{,}900$$

And $S = 2{,}570$

a. Find a two-sided 95% confidence interval for the true population mean.

$$21{,}225 \pm t_{14,0.025} \frac{2570}{\sqrt{16}} = 21{,}225 \pm 2.131 \cdot 642.5 = 21{,}225 \pm 1369$$

or

19,856 to 22,594

b. Consider the following hypothesis:

$$H_0: \quad \mu = 20{,}000$$
$$H_1: \quad \mu \neq 20{,}000$$
$$\alpha = 0.05$$

Can you reject the null hypothesis at this level of significance? **Briefly** explain your answer.

No, you cannot reject the null hypothesis since the 95% CI above includes 20,000.

Please place your name on
Each page now

NAME: ______________________________

Problem 2: **(25 percent)**

A new car is advertised as getting 32 miles per gallon in highway driving. The standard deviation of this sort of measurement is 1.5 miles per gallon.

a. Construct an appropriate test to determine whether or not the advertised mileage is correct at a level of significance of 0.05. **Justify whether a 1 or 2 sided test is appropriate.**

$$H_0: \quad \mu \geq 32$$
$$H_1: \quad \mu < 32$$
$$\alpha = 0.05$$

I would use a one sided test because as a consumer we are not worried about getting mileage that is better than the advertised rate, only about mileage that is worse than the advertised rate.

b. How large a sample would you need to take to detect a difference of 0.5 mile per gallon with a power of 80%?

$$n = \left(\frac{(z\alpha + z\beta)1.5}{0.5}\right)^2 = ((1.645+0.84)3)^2 = 55.57$$

so we should use $n = 56$

c. For this sample size, find the region in which you would reject the null hypothesis. If you did not get an answer to part (b) use a value of *n=50* (which is not the correct answer to part (b)).

$$\text{Reject if } \overline{X} \leq 32 - 1.645\left(\frac{1.5}{\sqrt{56}}\right) = 31.67$$

d. A sample of the size determined in part (b) is taken and the sample average is 31.62 miles per gallon. Can you reject the null hypothesis at a level of significance of 0.05?

Since 31.62<31.67, we reject the null hypothesis.

Please place your name on
Each page now

NAME: ______________________________

<u>Problem 3:</u> (30 percent)

The inner diameters of 20 washers are measured. The nominal diameter is supposed to be 1.25 cm. The following data were found:

$$\sum_{i=1}^{20} D_i = 25.4 \qquad \sum_{i=1}^{20} D_i^2 = 32.262275$$

a. Find the sample mean and the sample variance of the diameters.

$$\overline{D} = \frac{25.4}{20} = 1.27$$
$$s^2 = \frac{32.262275 - \left(25.4^2/20\right)}{19} = 0.000225$$

b. Find a lower 90% confidence interval for the variance of the diameters. If you did not get the answer to part (a) use a variance of 0.0003 (which is again **not** the correct answer to part (a)).

$$\sigma^2 \geq \frac{(n-1)s^2}{\chi^2_{19,0.01}} = \frac{(19)0.000225}{27.203} = 0.000157$$

c. If a washer is considered defective if its actual diameter is less than the nominal diameter, based on the data above, find the probability that a randomly selected washer is defective. (Again, if you did not get the answer to part (a) use a variance of 0.0003.)

An underlying assumption of the sort of work we are doing above (e.g., the confidence interval in (b)) is that the data come from a normal distribution. Thus, we can compute:

$$\begin{aligned} P\left(D < 1.25 \middle| \mu = 1.27\right) &= P\left(\frac{D-1.27}{0.015} \leq \frac{1.25-1.27}{0.015}\right) \\ &= P\left(z \leq -4/3\right) \\ &\cong 0.0918 - 0.3 \bullet 0.0017 \\ &= 0.09129 \end{aligned}$$

d. Using your answer to part (c), find the approximate probability that fewer than 75 washers out of 1,000 are defective?

$$
\begin{aligned}
P(\#\,\text{defective} < 75) &= P\left(\frac{\#\,\text{def} - 91.29}{\sqrt{82.956}} \le \frac{74.5 - 91.29}{\sqrt{82.956}}\right) \\
&= P(z \le -1.8434) \\
&\approx 0.0325
\end{aligned}
$$

<u>Problem 4:</u> (20 percent)

Data are collected on the GPAs of male undergraduates at Northwestern and female undergraduates at Northwestern. The sample can be summarized as follows:

	Men	Women
Average	2.96	3.17
Standard deviation	0.25	0.36
Sample size	9	16

a. Assuming that the variances of the men's GPAs and the women's GPAs are equal, test the null hypothesis that the GPA for men in the entire NU population is equal to GPA for women in the entire NU population against the alternative hypothesis that women have a higher GPA with α=0.1. **Clearly show your work**.

$$s^2_{pooled} = \frac{(8)0.25^2 + (15)0.36^2}{23} = 0.10626$$

$$t = \frac{3.17 - 2.96}{\sqrt{0.10626}\left(\sqrt{\frac{1}{9} + \frac{1}{16}}\right)} = 1.546$$

This has 23 degrees of freedom and the critical value at $\alpha = 0.1$ is 1.319

So we reject the null hypothesis.

b. Again, assuming that the true underlying variances are equal, find a two-sided 95% confidence interval for the true difference between women's GPAs and men's GPAs.

$$0.21 \pm 2.069 \cdot \sqrt{0.10626}\left(\sqrt{\frac{1}{9} + \frac{1}{16}}\right) = 0.21 \pm 0.281 = -0.071 \text{ to } 0.491$$