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IE 303 Statistics I
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Statistics I – Quiz
SOLUTIONS

Problem 1:

The Federal Aviation Administration just issued an order that 30% of the passengers boarding certain small aircraft are to be weighed. This is in response to the recent crash of a Beech 1900 aircraft that killed all 21 people aboard the aircraft. The FAA currently assumes the following weights for individuals

Person type	Winter	Summer
Adult	180	175
Child	80	80

These values include 20 pounds of carry on luggage for each adult.

Assume that the distribution of adult weights is Normal with a mean of 180 pounds and a standard deviation of 20 pounds.

a) If there are 21 adults on board the aircraft, what is the mean and variance of the total weight of these passengers?

Mean=21*180=3780 lb
Var = $21*(20)^2=21*400=8400$ lb^2

b) What is the distribution of the total weight?

Normal with a mean of 3780 and a variance of 8400

c) What is the probability that the total weight will exceed 3960 pounds?

$$
\begin{aligned}
P(X \geq 3960) &= P\left(\frac{X-3780}{\sqrt{8400}} \geq \frac{3960-3780}{\sqrt{8400}}\right) \\
&= P(Z \geq 1.96) \\
&= 0.025
\end{aligned}
$$

d) Find the value of the total weight that is exceeded with probability 0.005.

$$Weight = 3780 + 2.576\sqrt{8400} = 4016$$

e) Now assume that the plane has 18 adults and 6 children on board and that the average weight of a child is 80 pounds and the standard deviation of a child's weight is 15 pounds. Now what is the probability that the total weight will exceed 3960 pounds?

Mean = 18*180+6*80=3720 lb
Var = 18*(20)^2+6*(15)^2=8550 lb^2

$$\begin{aligned} P(X \geq 3960) &= P\left(\frac{X-3720}{\sqrt{8550}} \geq \frac{3960-3720}{\sqrt{8550}}\right) \\ &= P(Z \geq 2.595) \\ &\approx 0.005 \end{aligned}$$

<u>Problem 2:</u>

You work for an automobile manufacturer and you are interested in the number of miles that drivers can get from a set of brakes. You ask your dealers to record the vehicle mileage for customers who are replacing the **original** brakes on their vehicle. You obtain the following data (measured in thousands of miles) for 25 customers with brakes from a supplier that claims to produce brakes that last 60,000 miles with a standard deviation of 7500 miles. *These values – 60,000 and 7500 – are the theoretical or population values, **not** the sample values. The sample values are summarized below in the following two summations.*

$$\sum_{i=1}^{25} M_i = 1415 \qquad \sum_{i=1}^{25} M_i^2 = 81{,}625$$

where M_i is the mileage (in **thousands** of miles) of the ith customer.

a) Find the **sample** average brake life

$$\text{Average} = \frac{\sum_{i=1}^{25} 1415}{25} = 56.6 \text{ thousand miles}$$

b) Find the **sample** standard deviation of brake life

$$s^2 = \frac{\sum_{i=1}^{25} M_i^2 - \left(\sum_{i=1}^{25} M_i\right)^2 \Big/ 25}{24} = \frac{81625 - (1415)^2 / 25}{24} = 64$$

$$s = 8 \text{ thousand miles}$$

c) Assuming that brake life is Normally distributed and that the standard deviation of brake life is 7,500 miles, find a two-sided 95% confidence interval for the true mean tire life.
Note that this is ***NOT*** *exactly the standard deviation you should have obtained in part (b). That is a* ***sample*** *standard deviation. In this part and the subsequent parts of this question, use* ***7,500****, as the standard deviation, since that is the population value. The value in part (b) is a sample value and we do not yet know how to deal with sample standard deviations in confidence interval estimation and/or hypothesis testing in this class. That is to come soon….*

$$56.6 \pm 1.96\frac{7.5}{5} = 56.6 \pm 2.94$$

53.66 to 59.54

d) Test the following hypothesis (again assuming that brake life is Normally distributed with a standard deviation of 7,500 miles)

$$H_0: \quad \mu \geq 60{,}000$$
$$H_1: \quad \mu < 60{,}000$$

$$\alpha = 0.025$$

Can you reject the null hypothesis at this level of significance? In other words, do you have reasonable cause to believe that the brakes from the supplier are not meeting the claimed specifications? Briefly justify your answer.

We reject the null hypothesis since the confidence interval above (which has 0.025 probability in the right hand tail) does not include the value of 60,000.

Problem 3:

You have two methods of estimating the speed of traffic on a highway. The first method (method A) involves loop detection devices which are wire loops placed in the ground that detect the presence of a vehicle over the loop. Estimates are reported to a central computer every 30 seconds. The second approach (method B) involves automated video surveillance of the segment with a program attempting to assess the speed of the vehicles based on color changes in the image. In this case, estimates are reported to the computer every 40 seconds. Each method is clearly subject to error. Assume that each estimate from each method is independent of any other estimates by the same method or by the other method.

Every 4 minutes you need to report an estimate of the travel speed to radio station for broadcasting.

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Assume that method A has a standard deviation of 3 mph and method B has a standard deviation of 5 mph.

a) In 4 minutes you obtain 8 measurements from method A and 6 from method B. Find the variance and standard deviation of the unweighted sample **average** of the 14 measurements.

$$\bar{X} = \frac{1}{14}\left(X_1^A + X_2^A + X_3^A + X_4^A + X_5^A + X_6^A + X_7^A + X_8^A + X_1^B + X_2^B + X_3^B + X_4^B + X_5^B + X_6^B\right)$$

$$\begin{aligned} Var(\bar{X}) &= \left(\frac{1}{14}\right)^2 \left\{8Var\left(X_i^A\right) + 8Var\left(X_i^B\right)\right\} \\ &= \left(\frac{1}{14}\right)^2 \left\{8 \bullet 9 + 6 \bullet 25\right\} \\ &= 1.13265 \\ StDev(\bar{X}) &= \sqrt{1.13265} \\ &= 1.0643 \end{aligned}$$

b) Assuming the average speed reported by the 8 measurements using method A is 56 mph and by the 6 measurements using method B is 58.8 mph. What is the unweighted sample average of the 14 observations? *Hint: it is* ***not*** *57.4 mph which is the average of 56 and 58.8.*

$$\bar{X} = \frac{8 \bullet 56 + 6 \bullet 58.8}{14} = 57.2$$

c) Assuming the observations from each method are Normally distributed, find a 95 percent confidence interval for the true mean if you use the unweighted sample average as an estimator of the mean speed.

$$57.2 \pm 1.96 \bullet 1.0643 = 57.2 \pm 2.086$$
$$55.114 \text{ to } 59.286$$

d) Now, suppose you decide to weight the observations from method A more than the observations from method B since method A has a smaller standard deviation. Suppose you use weights of $w_A = 0.1$ and $w_B = \frac{0.1}{3}$. In particular, you use the following estimator:

$$\tilde{X} = 0.1\left(X_1^A + X_2^A + X_3^A + X_4^A + X_5^A + X_6^A + X_7^A + X_8^A\right) + \frac{0.1}{3}\left(X_1^B + X_2^B + X_3^B + X_4^B + X_5^B + X_6^B\right)$$

Assuming that each method is unbiased, is this an unbiased estimator of the true mean speed? Briefly justify your answer.

Yes because the weights add up to 1 (8*(0.1)+6*(0.1/3)=1).

e) Find the variance of $\tilde{X}$

$$\tilde{X} = 0.1\left(X_1^A + X_2^A + X_3^A + X_4^A + X_5^A + X_6^A + X_7^A + X_8^A\right) + \frac{0.1}{3}\left(X_1^B + X_2^B + X_3^B + X_4^B + X_5^B + X_6^B\right)$$

$$Var\left(\tilde{X}\right) = 0.01 \bullet 8 \bullet Var\left(X_i^A\right) + \frac{0.01}{9} \bullet 6 \bullet Var\left(X_i^B\right)$$

$$= 0.01 \bullet 8 \bullet 9 + \frac{0.01}{9} \bullet 6 \bullet 25$$

$$= 0.8866666$$

f) Again, assuming that the average speed reported by the 8 observations from method A is 56 mph and the average of the 6 observations from method B is 58.8 mph, find the value of $\tilde{X}$.

$$\tilde{X} = 0.1\left(X_1^A + X_2^A + X_3^A + X_4^A + X_5^A + X_6^A + X_7^A + X_8^A\right) + \frac{0.1}{3}\left(X_1^B + X_2^B + X_3^B + X_4^B + X_5^B + X_6^B\right)$$

$$\tilde{X} = 0.1 \bullet 8 \bullet 56 + \frac{0.1}{3} \bullet 6 \bullet 58.8$$

$$= 56.56$$

g) Find a 95 percent confidence interval for the population mean using $\tilde{X}$ as an estimator of the true mean.

$$56.56 \pm 1.96\sqrt{0.8866666} = 56.56 \pm 1.846$$

54.154 to 58.406