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IE 303 Statistics I | **Winter, 2002**
Mark S. Daskin | **Dept. of IE/MS** | **Northwestern University**

Statistics I

Quiz 1 – SOLUTIONS

Problem 1:

Due to heightened security concerns at airports, airlines are considering a two-phased screening process for all carry-on luggage. In the first phase, all bags would be X-rayed. This will flag certain bags as being potential threats while others will not be flagged. All flagged bags will then be subject to hand inspection and will be opened by security personnel.

Suppose the probability of being flagged at the first stage given that there is a dangerous substance (weapon, bomb, nail file, etc.) in your suitcase is 0.999 and the probability of being flagged given that there is **not** anything dangerous in your suitcase is 0.25. Further suppose 2.5% of the passengers are carrying something dangerous when they go through security.

a) Complete the following table putting the **joint** probability of both events in each of the four cells of the table. In other words, the top left hand cell should give the probability that a person has a dangerous item ***and*** the person is flagged for a second screening. ***(Report all numbers to 6 significant digits since some of the probabilities involved are very small.)***

	Person is flagged	**Person is NOT flagged**
Person IS carrying a dangerous item	**0.999*0.025 = 0.024975**	**0.001*0.025 = 0.000025**
Person is NOT carrying a dangerous item	**0.25*0.975 = 0.243750**	**0.75*0.975 = 0.731250**

b) What is the probability that a person will be flagged for the second stage of the screening? ***(Report the answer to 6 significant digits since some of the probabilities involved are very small.)***

0.268725

c) Given that someone is flagged for the second stage of the screening, what is the probability that they are carrying something dangerous? ***(Report the answer to 6 significant digits since some of the probabilities involved are very small.)***

$$P(dangerous|\,flagged) = \frac{P(dangerous \text{ and } flagged)}{P(flagged)} = \frac{0.024975}{0.268725} = 0.092939$$

d) Suppose that the second stage, hand inspection catches 98% of the individuals attempting to carry dangerous items onto a plane. In other words, the probability of being caught ***given*** that you are trying to carry something dangerous onto the plane ***and*** you were flagged in the initial screening is 0.98. Find the probability that someone is not caught ***and*** that they are carrying something dangerous.

Note that this can be viewed as the following:

$$P(\text{not caught AND dangerous})$$

$$= \begin{Bmatrix} P(\text{not caught AND dangerous AND flagged}) + \\ P(\text{not caught AND dangerous AND NOT flagged}) \end{Bmatrix}$$

$$= \begin{Bmatrix} P(\text{not caught}\,|\,\text{dangerous AND flagged})P(\text{dangerous AND flagged}) + \\ P(\text{not caught}\,|\,\text{dangerous AND NOT flagged})P(\text{dangerous AND NOT flagged}) \end{Bmatrix}$$

You should have enough information to compute this joint probability using the last equation above.

$$P(\text{not caught AND dangerous})$$

$$= \begin{Bmatrix} P(\text{not caught AND dangerous AND flagged}) + \\ P(\text{not caught AND dangerous AND NOT flagged}) \end{Bmatrix}$$

$$= \begin{Bmatrix} P(\text{not caught}\,|\,\text{dangerous AND flagged})P(\text{dangerous AND flagged}) + \\ P(\text{not caught}\,|\,\text{dangerous AND NOT flagged})P(\text{dangerous AND NOT flagged}) \end{Bmatrix}$$

$$= (0.02)(0.024975) + (1)(0.000025) = 0.000525$$

e) Suppose 50,000 people are screened every day through such a security system. What is the expected number of people carrying something dangerous who are not caught? This involves treating the probability you computed in part (d) as the probability of "failure" in a series of 50,000 Bernoulli trials and we are asking for the expected number of failures.

$$50{,}000(0.000525) = 26.225$$

f) Again, assuming that 50,000 people are screened, what is the probability that **all** people carrying something dangerous will be caught? In other words, what is the probability of NO failures?

$$(1-0.000525)^{50{,}000} = 0.999476^{50{,}000} = 4.05 \bullet 10^{-12}$$

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Problem 2: (30 Percent)

Ten samples of 16-ounce cans of tomato soup are tested to determine the actual quantity of soup in the cans. The following data are recorded:

16.03	16.08	16.11	16.01
16.04	15.92	15.98	15.97
16.07	16.09		

Sorted: 15.92 15.97 15.98 16.01 16.03 16.04 16.07 16.08 16.09 16.11

a) Find the sample average

$$\bar{X} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{160.3}{10} = 16.03$$

b) Find the sample variance

$$s^2 = \frac{\sum_{i=1}^{10} x_i^2 - 10 \bullet 16.03^2}{9} = \frac{2569.642 - 2569.609}{9} = 0.003644$$

c) Find the sample median

$$\text{Median} = \frac{16.03 + 16.04}{2} = 16.035$$

d) Find the first quartile (i.e., the value of Q_1) as well as the value of the third quartile (i.e., the value of Q_3).

$$Q_1 = 15.97 + (2.75 - 2)(15.98 - 15.97) = 15.9775$$
$$Q_3 = 16.08 + (8.25 - 8)(16.09 - 16.08) = 16.0825$$

e) Find the range of the data

$$Range = 16.11 - 15.92 = 0.19$$

f) Find the intequartile range.

$$IQR = 16.0825 - 15.9775 = 0.105$$

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Problem 3: (40 Percent)

100 lightbulbs (of with 50 are 60 watt bulbs and 50 are 100 watt bulbs) are tested until they burn out. The number of hours that they last is tabulated. The data are summarized below:

60 watts: $\sum_{i=1}^{50} t_i = 46{,}916$ $\sum_{i=1}^{50} t_i^2 = 59{,}906{,}454$

100 watts: $\sum_{i=1}^{50} t_i = 59{,}439$ $\sum_{i=1}^{50} t_i^2 = 96{,}678{,}495$

where t_i is the number of hours that bulb i lasts until it burns out.

a) Find the sample mean average bulb life for each wattage.

60 watt: $\overline{X} = \dfrac{46916}{50} = 938.32$

100 watt $\overline{X} = \dfrac{59439}{50} = 1188.78$

b) Find the sample variance of the bulb life for each wattage.

60 watt $s^2 = \dfrac{59906454 - (46916 \bullet 46916/50)}{49} = 324{,}168.018$

100 watt $s^2 = \dfrac{96678494 - (59439 \bullet 59439/50)}{49} = 530{,}991.869$

c) Find the sample standard deviation of the bulb life for each wattage.

60 watt $s = 569.36$

100 watt $s = 728.69$

d) Find the sample mean, variance and standard deviation for all 100 bulbs taken together.

$$\overline{X} = \frac{106355}{100} = 1063.55$$

$$s^2 = \frac{156584950 - (106355 \bullet 106355/100)}{99} = 439{,}101.92$$

$$s = 662.65$$

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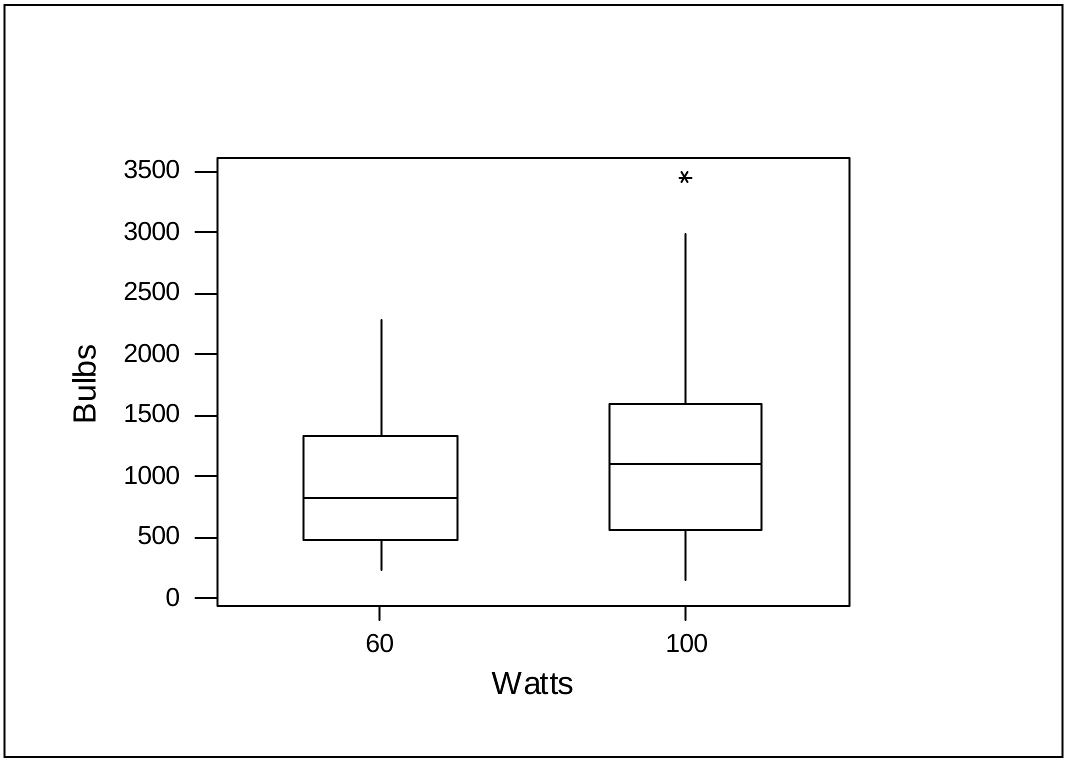
The boxplot for the data is shown below. In this plot we have separated the data into the 50 observations that were made on 60 watt bulbs and the 50 observations made on 100 watt bulbs.



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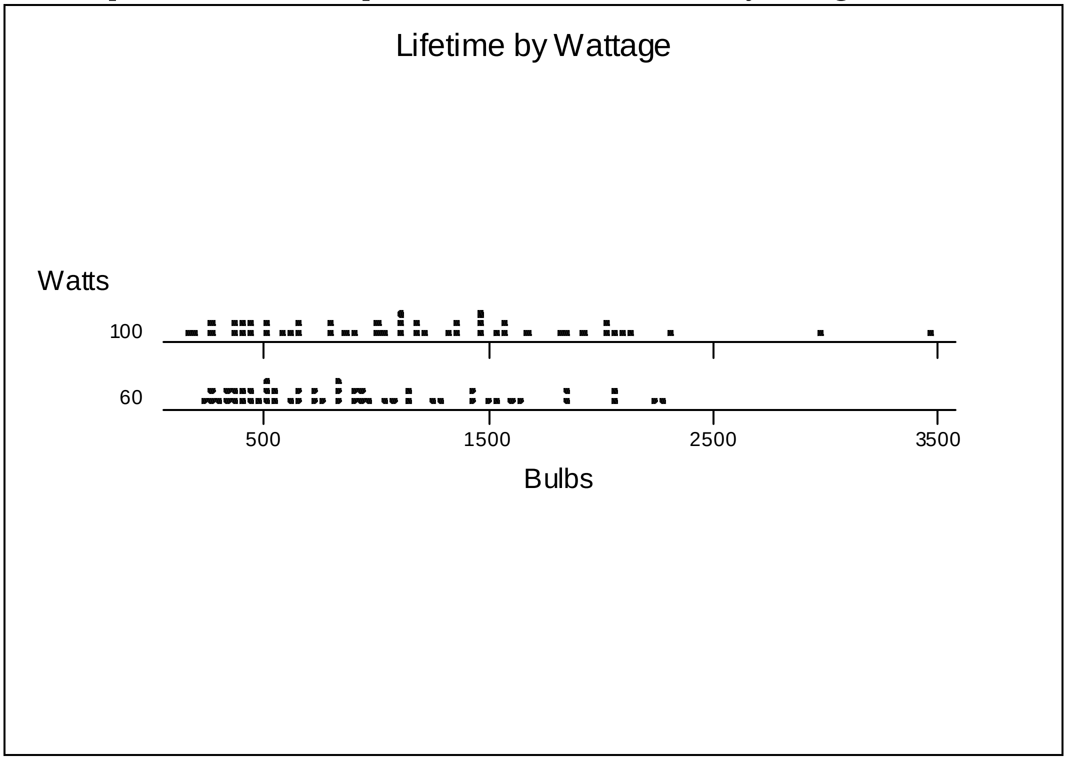
The plot below is the dot plot of the actual observations by wattage.



Estimate the sample median of each sample (60 watt and 100 watt) from the plots above.

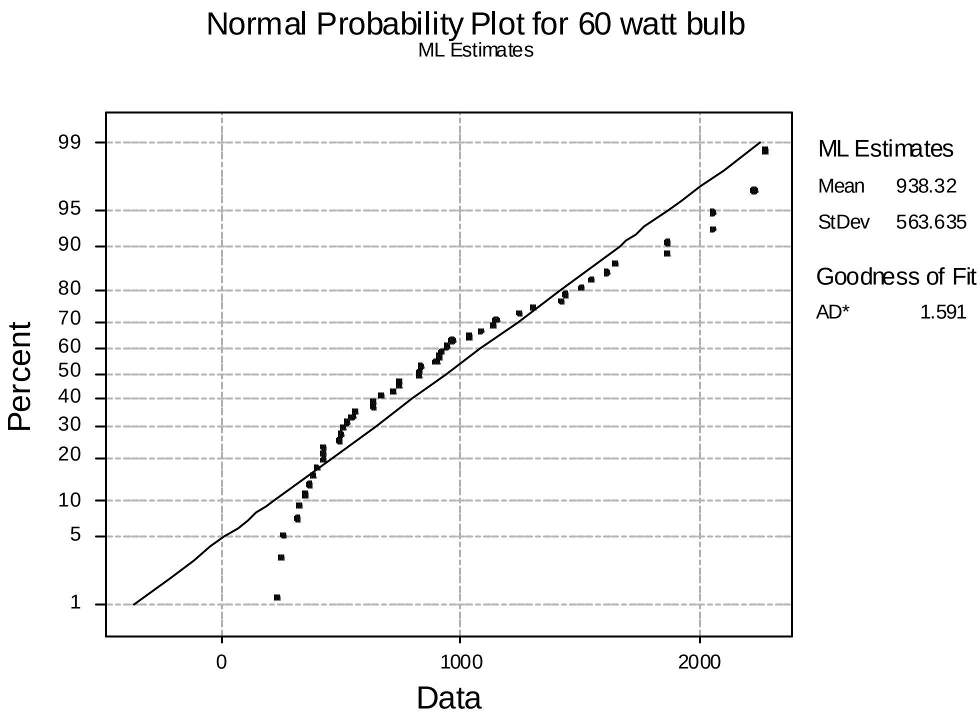
60 watt: median = 823
100 watt median=1103

Clearly I did not expect this exact an answer. I have the complete raw data. If you were close you would get credit.

e) Do the data appear to be skewed of symmetric? If the observations are skewed are they skewed to the right or to the left?

Both datasets appear to be skewed to the right.

f) The figure below shows the Normal Probability Plot for 60 watt



Do the data appear to be Normally distributed? If not, what sort of transformation might make the transformed data Normally distributed?

The data do not appear to be Normally distributed. This confirms that the data are skewed to the right. A log transform might well result in transformed observations that are closer to being Normally distributed.