 Name ____________________

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IE 202 – Introduction to Probability
Quiz 1 -- SOLUTIUONS

Problem 1 (30 percent):

The elite units of the Israeli army select 90% of their recruits based on rigorous emotional, physical and psychological testing; the other 10% are selected at random from those who would normally be rejected. The theory behind this is, apparently, that all the extensive testing is not foolproof and that this method gives the army a chance at some potentially good recruits that they might otherwise reject.

Suppose that 40% of those selected through the testing method make it through the training (i.e., do not drop out) and are in such a unit 1 year later. Also suppose that 15% of those selected at random make it through the training and are in an elite unit a year later.

You might find the following table useful in answering the rest of this question:

	Selection Method	
Make it through training	**Testing**	**Random**
Yes		
No		

	Selection Method		
Make it through training	**Testing**	**Random**	**Marginal**
Yes	**0.36**	**0.015**	**0.375**
No	**0.54**	**0.085**	**0.625**
Marginal	0.9	0.1	**1.000**

a) What is the probability that a soldier was selected based on the testing and that he or she survives a year?

This probability is 0.36.

b) What is the probability that a soldier was selected at random and that he or she survives a year?

This probability is 0.015.

c) What is the probability that a soldier makes it through the training and is in the unit a year later?

This probability is 0.375.

d) Given that a soldier has made it through the training and is in the unit a year later, what is the probability that he or she was originally selected based on the testing?

This is 0.36/0.375 or 0.96

e) Given that a soldier did **not** make it through the training, what is the probability that he or she was selected based on the testing?

This is 0.54/0.625 or 0.864

f) Is the survival probability independent of the selection method? Briefly justify your answer.

No, they are clearly not independent. If they were independent, we would expect that the joint probability of having been selected at random, for example and making it through the training (which is 0.015) would be equal to the product of the marginal probabilities (0.375x0.1 = 0.0375). Since these two values are not equal, it is clear that the survival probability is not independent of the selection method. In fact, the problem statement indicated that the survival probability depended on the selection method.

Problem 2 (35 percent):

On Friday afternoon, the IE faculty have "Cool Guys" lunch. What this means is that we try to go to lunch together on Fridays. "Guy" is a generic and non-gender specific term!

Of the faculty members in the department, some almost never go to lunch and so we will not consider them for the purpose of this problem. There are 13 faculty members who generally participate in Cool Guys: Ankenman, Apley, Daskin, Fourer, Hazen, Homem-de-Mello, Hopp, Iravani, Linetsky, Mehrotra, Nelson, Smilowitz, d Staum.

a) Assuming that there is a probability of 0.7 of a faculty member being able to attend Cool Guys (and each professor's ability to attend is independent of that of every other professor) what is the probability that **no one** will be able to attend Cool Guys Lunch (CGL) on any given Friday?

P(No one can attend) = $(0.3)^{13}$ = 1.59 x 10^{-7}.f

b) Find the probability of there being 8 people who go to lunch together on some Friday?

$$P(8\text{ people}) = \binom{13}{8} 0.7^8 0.3^5 = 0.1803$$

c) How many different ways are there of getting 8 different faculty members for lunch?

$$\text{Number of different ways of getting 8 people out of 13} = \binom{13}{8} = 1287$$

d) Four of the 13 faculty members above are from the Production and Logistics group (Daskin, Hopp, Iravani, and Smilowitz). Assuming there are 8 faculty members going to lunch, what is the probability that there will be no faculty from the Production and Logistics group?

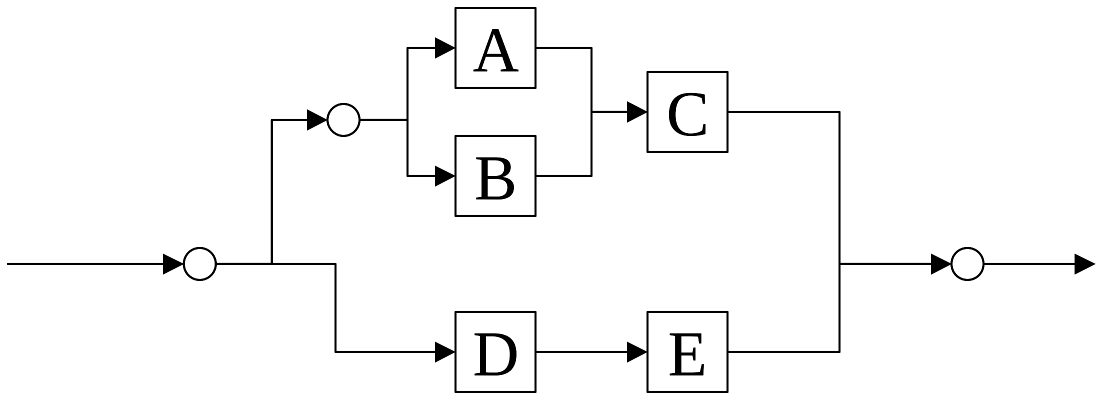
$$P(\text{none of the 8 are from P \& L group}) = \frac{\binom{4}{0}\binom{9}{8}}{\binom{13}{8}} = \frac{1 \bullet 9}{1287} = 0.006993$$

e) Again, assuming that 8 faculty go to lunch, what is the probability of having exactly 2 faculty members from the esteemed Production and Logistics group, the "coolest" of the Cool Guys?

$$P(\text{2 of the 8 are from P \& L group}) = \frac{\binom{4}{2}\binom{9}{6}}{\binom{13}{8}} = \frac{6 \bullet 84}{1287} = 0.3916$$

Problem 3 (35 percent):

Consider the following electrical system:



The system will work if either the top path through components A, B and C works or if the bottom path through components D and E works. The top path will work if (1) either component A or B works or both and (2) component C works. The bottom path will work if both components D and E work.

The component reliabilities are shown in the table below.

Component	Reliability
A	0.7
B	0.7
C	0.85
D	0.95
E	0.9

a) What is the probability that the subsystem composed of components A and B will work?

P(subsystem with A and B will work) = 1-P(A fails and B fails)
= 1-(0.3)(0.3)
= 1-0.09
= 0.91

b) What is the probability that the top path will work?

P(top path will work) = P(subsystem with A,B works AND C works)
= P(subsystem with A,B works)P(C works)
= (0.91)(0.85)
= 0.7735

c) What is the probability that the bottom path will work?

P(bottom path will work) = P(D works AND C works)
= P(D works)P(C works)
= (0.95)(0.9)
= 0.855

d) What is the probability that the system will work?

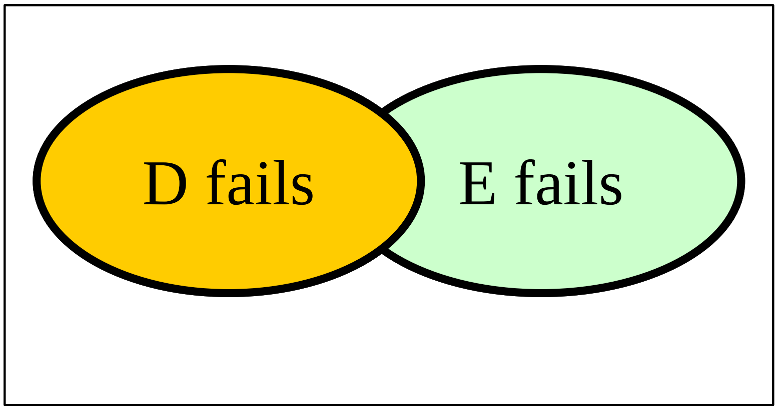
P(System works) = 1-P(top fails AND bottom fails)
= 1-P(top fails)P(bottom fails)
= 1-(1-0.7735)(1-0.855)
= 0.9672

e) What is the probability that the bottom path will fail?

This is just 1-P(bottom works) = 1-0.855 = 0.145

f) *HINT: For the next 4 parts of the problem, you may want to draw a Venn diagram showing components D and E failing.* Given that the bottom path has failed, what is the probability that component D has failed? Notice that the probability that the bottom path fails AND component D fails is simply the probability that component D fails.

Note that the bottom fails if **either** D fails or E fails



P(D fails | bottom fails) = P(D fails and Bottom fails)/P(bottom fails)
= P(D fails)/P(bottom fails)
= 0.05/0.145
= 0.34483

g) Given that the bottom path has failed, what is the probability that component E has failed?

P(E fails | bottom fails) = P(E fails and Bottom fails)/P(bottom fails)
= P(E fails)/P(bottom fails)
= 0.1/0.145
= 0.68966

h) Given that the bottom path has failed, what is the probability that **both** component D and component E have failed?

P(both fail | bottom fails) = P(Both and Bottom fails)/P(bottom fails)
= P(D fails AND E fails)/P(bottom fails)
= (0.1)(0.05)/0.145
= 0.03448

i) Given that the bottom path has failed, what is the probability that ONLY component D has failed?

P(ONLY D fails | bottom fails)
= P(D fails and Bottom fails)/P(bottom fails) – P(Both | bottom)
= 0.34483--0.03448
= 0.31034

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Work Sheet