IE 202 – Introduction to Probability
Quiz 2 – SOLUTIONS

Note that the last page is a table of Normal distribution values like Table A.3 in the text in case you need it.

Problem 1: (25 percent)

Assume that major crimes occur in a county occur according to a Poisson process. The rate of robberies is 17 per day. The murder rate is 0.67 per day. (While this may seem like a very high murder rate, these are the actual rates for Washington DC, computed over the period from 1960 through 2000. Welcome to our nation's capital!) Assume that the processes are independent.

a) What is the mean number of robberies over the course of a 7-day week?

17*7 = 119

b) What is the variance of the number of robberies over the course of a week?

17*7 = 119

c) What is the mean number of murders in a 7-day week?

0.67*7 = 4.69

d) What is the probability of there being **no** murders in a week?

$e^{-4.69} = 0.009187$

e) What is the probability of there being exactly 5 murders in a week?

$$\frac{\left(4.69^5\right)e^{-4.69}}{5!} = 0.1737$$

f) What is the probability of there being more than 5 murders?

$$1 - \sum_{k=0}^{5} \frac{4.69^k e^{-4.69}}{k!} = 0.3298$$

<u>Problem 2: 10 Percent *(Food for thought)*</u>

The population of the United States is about 300 million people. Actually, we will hit that number sometime this summer. Many characteristics of people – height, weight, intelligence tests (for what they are worth), views on political issues, income, physical or sports capabilities – tend to be approximately Normally distributed.

Being 3 standard deviations from the mean of a normal distribution is viewed as being quite deviant or extreme in a statistical or probabilistic sense. With 300 million people in the US, about how many are 3 or more standard deviations away from the mean of whatever is being measured assuming the characteristic follows a Normal distribution?

$(300{,}000{,}000)(1 - 0.99865)2 = 810{,}000$ **or almost 1 million people**

Problem 3: (20 Percent)

A computer screen is composed of millions of tiny pixels. Sometimes a pixel will be defective (as is a pixel on my home computer screen).

The probability that a pixel will be defective is $2\text{x}10^{-7}$. Assume that whether or not any individual pixel works is *independent* of the probability of another pixel on the same screen working. A screen has a resolution that is 1600x1200 and so it is composed of 1,920,000 pixels.

a) Find the probability that a screen will have NO defective pixels.

You can use a Poisson approximation to the Binomial with $\lambda = np = (1920000)2 \bullet 10^{-7} = 0.384$, so the probability is $e^{-0.384} = 0.681131$. This is the same value that you would obtain if you used $(1-2\bullet 10^{-7})^{1920000}$ from the Binomial

b) Find the probability that a screen will have exactly ONE defective pixel.

Again using the Poisson approximation we compute $\dfrac{0.384e^{-0.384}}{1!} = 0.261554$.

Using the exact Binomial expression we would get 0.261555.

c) A screen is deemed defective if two or more pixels are defective. What is the probability that a screen will be defective?

1-0.681131-0.261554=0.057134

d) Screens are made in Singapore and shipped to the US in batches of 250 at a time. What is the (approximate) probability that the number of defective screens in a batch of 250 is 20 or less?

Here you should use the Normal approximation to the Binomial with n=250 and p=0.057134. This gives a mean for the Normal of 14.329 and a variance of 13.507. The probability of 20 or less is then

$\Phi\left(\dfrac{20.5-14.329}{\sqrt{13.5073}}\right) = \Phi(1.679) = 0.9535$. **If you approximate this to**

$\Phi(1.645) = 0.95$ **that is fine as well.**

Problem 4: (25 Percent)

Washington University, in an effort to increase its apparent selectivity (which is the ratio of the number of accepted students to the number of applicants and which is a measure used by such ranking "services" as *US News and World Report*), has taken to mailing high school students **tons** of information. The goal is to induce mores students to apply, thereby increasing the denominator.

A (hypothetical) high school statistics class collected data on this and found that the number of mailings a student received from Wash U in her junior year followed a Poisson distribution with $\lambda = 0.75$ mailings per week.

a) What are the mean and variance of the number of mailings a student can expect over the course of a 40-week junior year?

Mean = variance = 40*0.75 = 30

b) What is the probability (approximate if necessary) that a student will receive more than 35 mailings during her junior year?

Use a Normal approximation to the Binomial with a mean of 30 and standard deviation of the square root of 30 or 5.477. This means that we want $1-\Phi\left(\frac{35.5-30}{5.477}\right)=1-\Phi(1.004)=1-0.8413=0.1587$

c) What is the approximate probability that a student will receive less than 20 mailings during her junior year?

Use a Normal approximation to the Binomial with a mean of 30 and standard deviation of the square root of 30 or 5.477. This means that we want $\Phi\left(\frac{19.5-30}{5.477}\right)=\Phi(-1.917)=1-0.9582=0.0276$**. If you approximated this as** $\Phi\left(\frac{19.5.5-30}{5.477}\right)=\Phi(-1.917)\approx\Phi(-1.96)=0.025$**, I would accept that as well.**

d) What is the probability that it will be 3 weeks or more between two successive mailings that a student receives?

This is the probability of NO Poisson events in a 3 week period, when the parameter of the Poisson distribution is 0.75*2.25 or 2.25. This is

$e^{-2.25} = 0.1054$

e) What is the probability that the time between the start of the junior year and when a student receives her 4th mailing from Wash U is more than 6 weeks.

This is the probability of 3 or fewer Poisson events in 6 weeks, where the parameter of the Poisson distribution is 0.75*6 = 4.5. Thus we want

$$\sum_{k=0}^{3} \frac{4.5^k e^{-4.5}}{k!} = 0.3423$$

Problem 5: (20 percent)

The time **in days** between house fires in a small rural community has been found to follow an exponential distribution with parameter $\mu = 0.025$.

a) What is the **mean** time in days between fires? What is the **standard deviation** of the time between fires?

$mean = \frac{1}{\mu} = 40$ **which is also the standard deviation of the time between fires.**

b) What is the probability that the time between the first of the year (Jan 1) and the third home fire is 180 days or less.

This is 1 minus the probability that there are 2 or fewer Poisson events during a 180 day time period where the parameter of the Poisson is $180\mu = 4.5$.

Thus we want $1 - \sum_{k=0}^{2} \frac{4.5^k e^{-4.5}}{k!} = 0.8264$

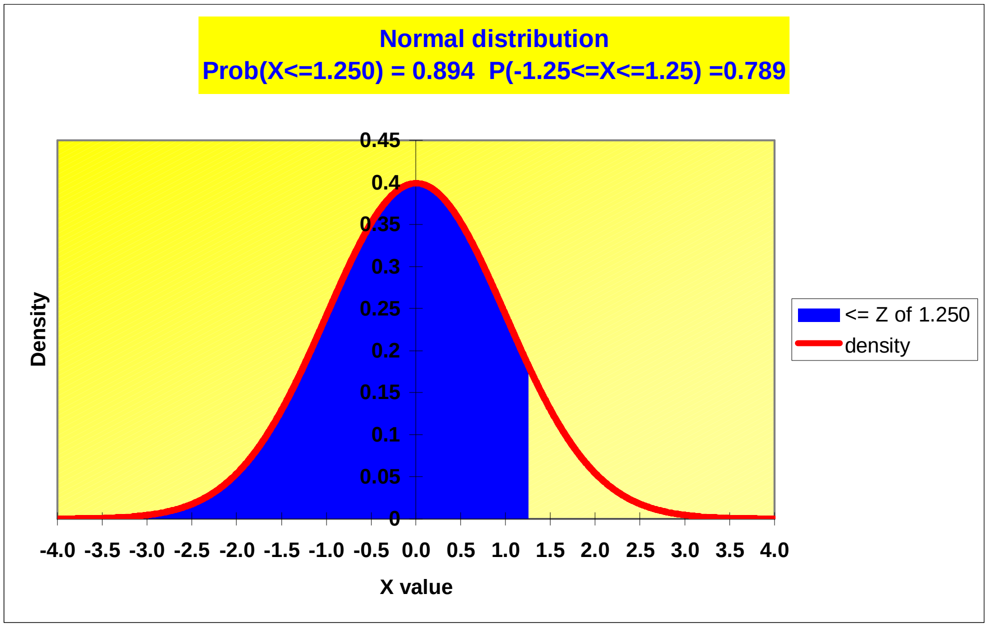
c) The fire chief is very concerned about the rate of residential fires and so he initiates a public education program. In the 180 days (six months) after the end of the program, there were only 2 home fires. He argues that the program was successful and asks the town council to approve such a program on an annual basis. What is the probability of having 2 or fewer fires in a 180 day period *if the program had no effect whatsoever*? Based on this, do you think there is sufficient evidence to warrant making the program a permanent fixture of the community? **Show the probability and then briefly state whether you think the fire chief has a convincing argument for perpetuating the public education program.**

The probability is 1 minus the probability computed in part (b) or 0.1736. This is about 1/6 which is, in my view, too large to justify this sort of public expenditure. Would we justify a public expenditure like this by rolling a die and seeing if it came up as a 1? If not, then this is too high a probability.

Name ____________________________________

Please place your name on all pages NOW

Work Page



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	**0.5**	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.55567	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.57926	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.60642	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.6293	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705402	0.70884	0.71226	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.77035	0.773373	0.776373	0.77935	0.782305	0.785236
0.8	0.788145	0.79103	0.793892	0.796731	0.799546	0.802338	0.805106	0.80785	0.81057	0.813267
0.9	0.81594	0.818589	0.821214	0.823814	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	**0.841345**	0.843752	0.846136	0.848495	0.85083	0.853141	0.855428	0.85769	0.859929	0.862143
1.1	0.864334	0.8665	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881	0.882977
1.2	0.88493	0.88686	0.888767	0.890651	0.892512	0.89435	0.896165	0.897958	**0.899727**	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914656	0.916207	0.917736
1.4	0.919243	0.92073	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.93822	0.939429	0.94062	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	**0.949497**	**0.950529**	0.951543	0.95254	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.96407	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971284	0.971933	0.972571	0.973197	0.97381	0.974412	**0.975002**	0.975581	0.976148	0.976705
2.0	0.97725	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.98983	**0.990097**	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.99224	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.99379	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	**0.994915**	**0.99506**	0.995201
2.6	0.995339	0.995473	0.995603	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.99702	0.99711	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.99825	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	**0.99865**	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.99893	0.998965	0.998999
3.1	0.999032	0.999064	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.99955	0.999566	0.999581	0.999596	0.99961	0.999624	0.999638	0.99965
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.99972	0.99973	0.99974	0.999749	0.999758